香港城市大學
City University of Hong Kong

Algo Trading and Technical Analysis

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Outline

- ❑ Technical Analysis
- ❑ Wavelet Transform and Multi-resolution Moving Average

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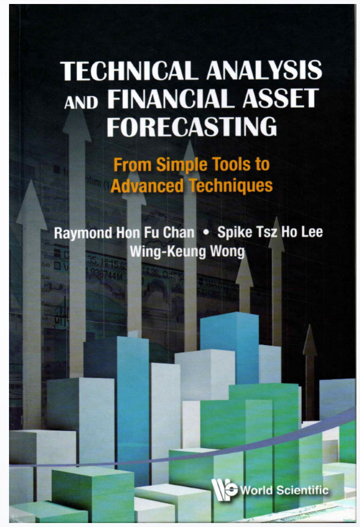
Algo Trading

- Execute buy/sell orders using automated pre-programmed trading instructions based on time, price, and volume.
- Used by investment banks, pension funds, mutual funds, and hedge funds
- In 2019, around 92% of Forex market tradings were performed by algorithms rather than humans
- What are the trading instructions?



Material based on the book

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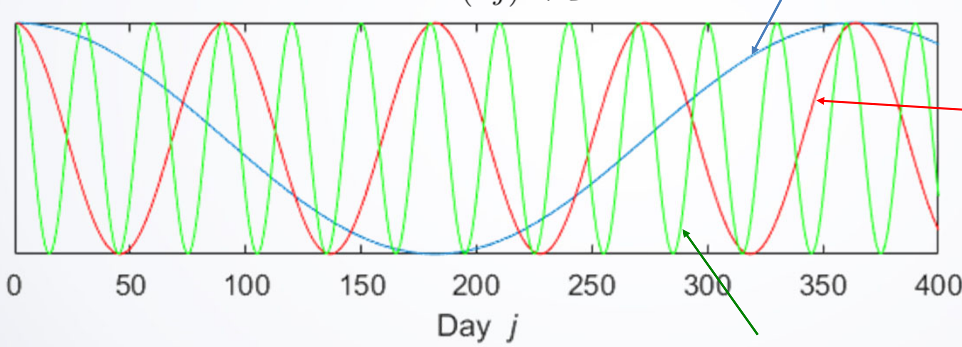
Frequency of a Signal

Yearly, seasonal, and monthly periodic signals

$\cos(\omega j) \to e^{\iota j\omega}$

$\cos(2\pi j/364)$ $\omega = \frac{2\pi}{364}$ low frequency



$\cos(2\pi j/91)$ $\omega = \frac{2\pi}{91}$ mid frequency

$\cos(2\pi j/30)$ $\omega = \frac{2\pi}{30}$ higher frequency

Slowly moving functions have lower frequencies.

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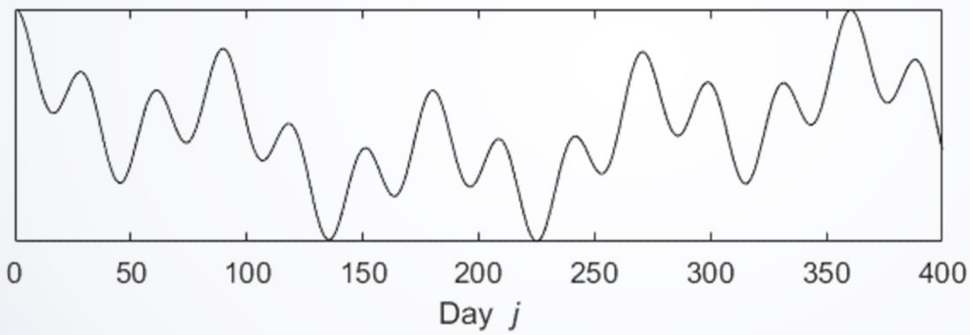
Signal from Component Functions

Combined effect of the signals $f_y(t) + f_s(t) + f_m(t)$



Does not look like a stock price chart

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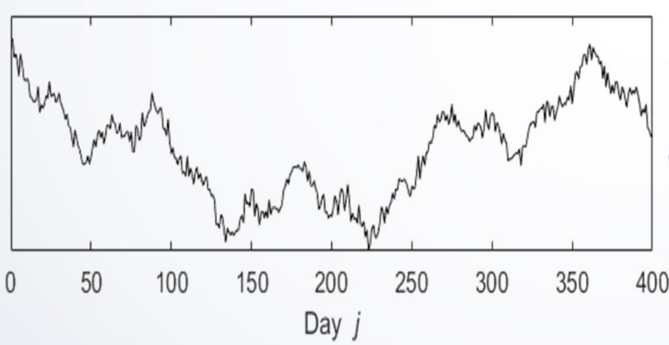
Analyzing a Signal

analysis

$8f_y(t) + 4f_s(t) + 2f_m(t) + f_d(t) + \mathcal{N}(0,1)$



Hang Seng Index (HSI) 2007–2011

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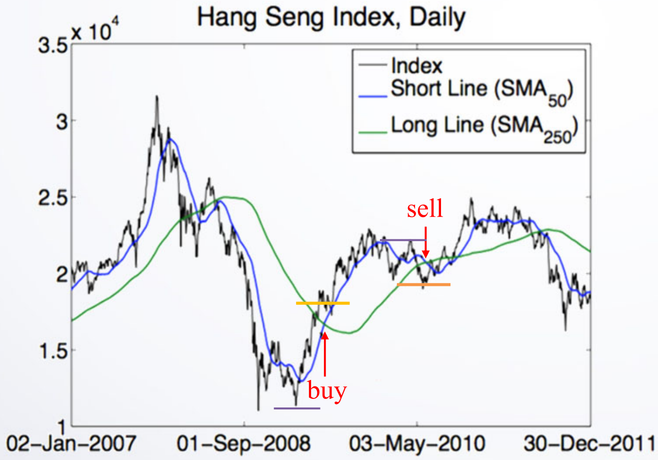
Classical Technical Analysis

Simple moving average:

$$\text{SMA}_M = \frac{x_0 + x_{-1} + \cdots + x_{1-M}}{M}$$



- x_0 today's price
- x_{-1} yesterday's price
- short line (smaller M) dynamic (some noise)
- long line (bigger M) smoother (less noise)
- buy when short ↑ long $\text{SMA}_{50} > \text{SMA}_{250}$
- sell when short ↓ long $\text{SMA}_{50} < \text{SMA}_{250}$

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Prediction Power

Why there is a lag?

$$\text{SMA}_5\colon\ y_j = \frac{x_j + x_{j-1} + x_{j-2} + x_{j-3} + x_{j-4}}{5} = \sum_{k=0}^{4} \frac{1}{5} x_{j-k}$$

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Moving Averages as Linear Filters

SMA can be written in convolutional form:

$$y_j = \sum_{k=0}^{\infty} h_k x_{j-k},$$

- ☐ $\mathbf{x} = (\ldots, x_{-2}, x_{-1}, x_0, x_1, x_2, \ldots)$: stock prices (0 is today)
- ☐ $\mathbf{h} = (\ldots, h_{-2}, h_{-1}, h_0, h_1, h_2, \ldots)$: filter tabs
- ☐ $\mathbf{y} = (\ldots, y_{-2}, y_{-1}, y_0, y_1, y_2, \ldots)$: indicator ($y_0$ is today's indicator)

For SMA_M: $h_j = \frac{1}{M}$, for $0 \le j < M$ and 0 otherwise.

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Frequency Response

Consider the stock price that has only one frequency ω, i.e. $x_j = e^{\iota j\omega}$:

$$y_j = \sum_{k=0}^{\infty} h_k x_{j-k} = \sum_{k=0}^{\infty} h_k e^{\iota(j-k)\omega} = \left\{\sum_{k=0}^{\infty} h_k e^{-\iota k\omega}\right\} e^{\iota j\omega} \equiv H(\omega)e^{\iota j\omega} = H(\omega)x_j.$$

- ☐ y_j is just a multiple of x_j by $H(\omega)$
- ☐ $H(\omega) = |H(\omega)|e^{\iota\phi(\omega)}$: frequency response function

$$y_j = H(\omega)x_j = H(\omega)e^{\iota j\omega} = |H(\omega)|e^{\iota\phi(\omega)}e^{\iota j\omega} = |H(\omega)|e^{\iota(j\omega+\phi(\omega))} = |H(\omega)|e^{\iota\omega(j+\phi(\omega)/\omega)}$$

- ☐ $\phi(\omega)$: phase shift—time lag ($\phi(\omega) < 0$) or time lead ($\phi(\omega) > 0$)
- ☐ $|H(\omega)|$: gain function—which frequencies are damped

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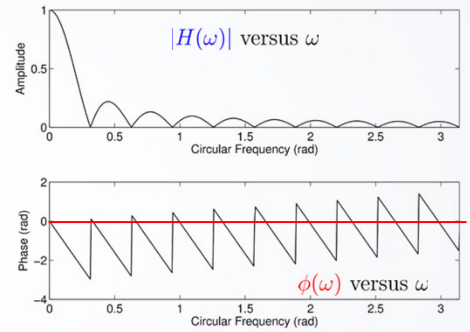
Simple Moving Average

$$\text{SMA}_M = y_j = \frac{x_j + x_{j-1} + \cdots + x_{j-M+1}}{M}$$

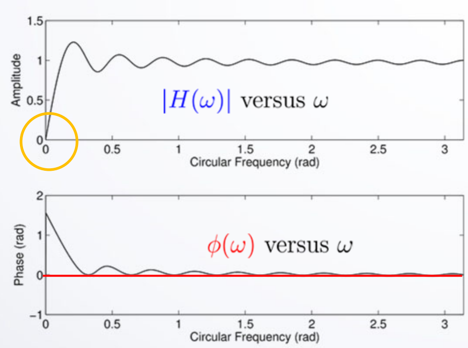
$$y_j = |H(\omega)| e^{\iota(j\omega + \phi(\omega))}$$



- ☐ $|H(\omega)|$: low frequency kept, high frequency damped (smooth output signal)
- ☐ $\phi(\omega) < 0$ in low frequency: time lag in response

Practical Indicators

- ☐ Difference of smoothing indices: e.g., $\text{SMA}_{50} - \text{SMA}_{250}$
- ☐ $\phi(\omega) \geq 0$ for some ω: leading in time for some frequencies
- ☐ $|H(\omega)|$: damp low frequency, kept high frequency (noisier)

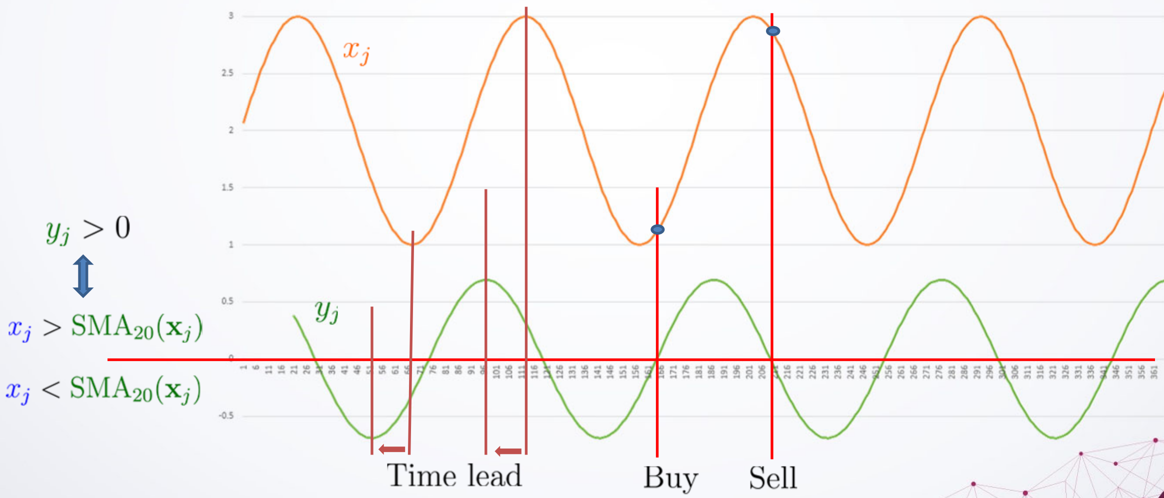


$I - \text{SMA}_{20}$

- ☐ $y_j = \text{SMA}_1(\mathbf{x}_j) - \text{SMA}_{20}(\mathbf{x}_j) = x_j - \text{SMA}_{20}(\mathbf{x}_j)$
- ☐ $\phi(\omega) \geq 0$: always leading in time
- ☐ $|H(\omega)| \approx 1$ when $\omega > 0.1$

In Theory: Noiseless Case

- ❑ Ideal seasonal stock price: $x_j = 2 + \sin(\frac{2\pi j}{91})$
- ❑ Indicator: $y_j = x_j - \text{SMA}_{20}(\mathbf{x}_j)$



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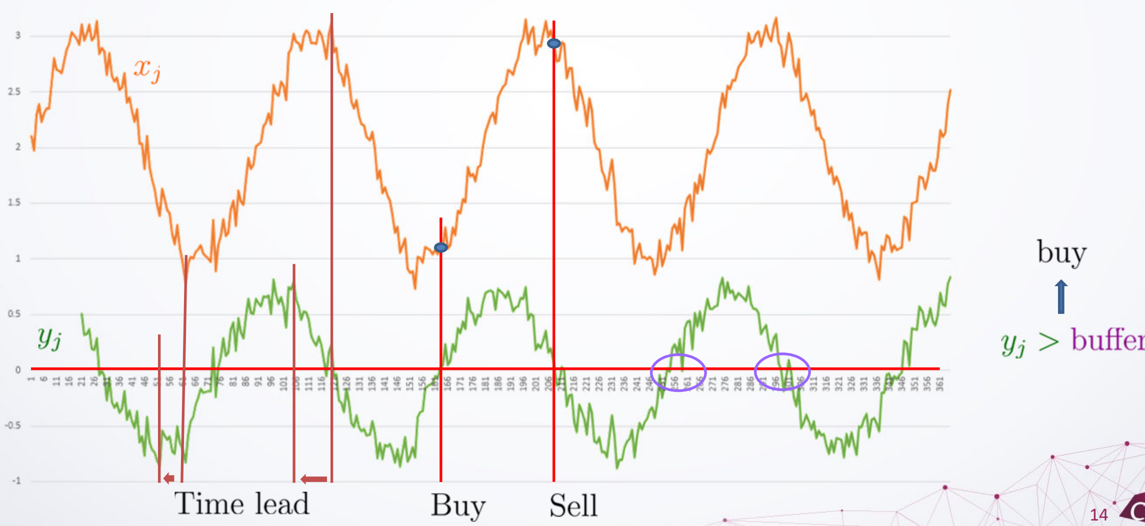
In Theory: Small Noise Case

- ❑ Seasonal stock price: $x_j = 2 + \sin(\frac{2\pi j}{91}) + \frac{1}{10}\mathcal{N}(0,1)$
- ❑ Indicator: $y_j = x_j - \text{SMA}_{20}(\mathbf{x}_j)$



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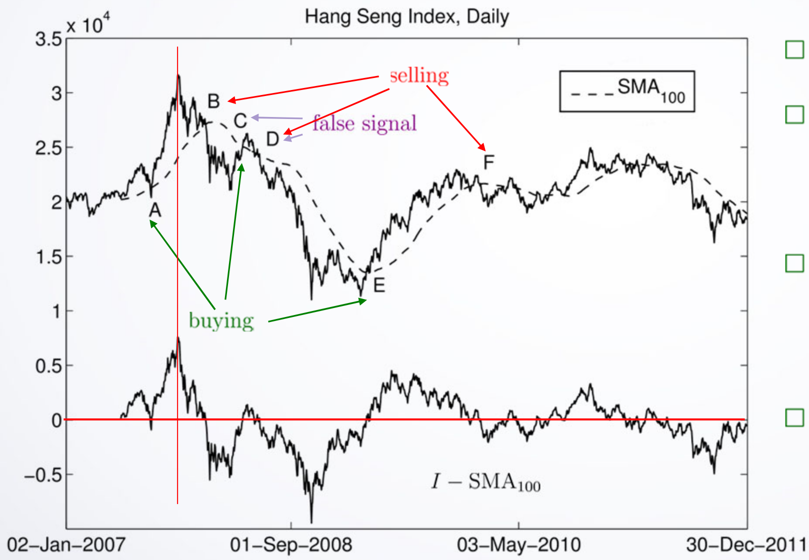
Does It Work in Practice?



- buy once $x_j > \mathrm{SMA}_{100}(\mathbf{x}_j)$
- sell once $x_j < \mathrm{SMA}_{100}(\mathbf{x}_j)$

⬇

- buy once

  $$x_j > \mathrm{SMA}_{100}(\mathbf{x}_j)(1 + p\%)$$

- sell once

  $$x_j < \mathrm{SMA}_{100}(\mathbf{x}_j)(1 - p\%)$$

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Outline

- Technical Analysis
- Wavelet Transform and Multi-resolution Moving Average

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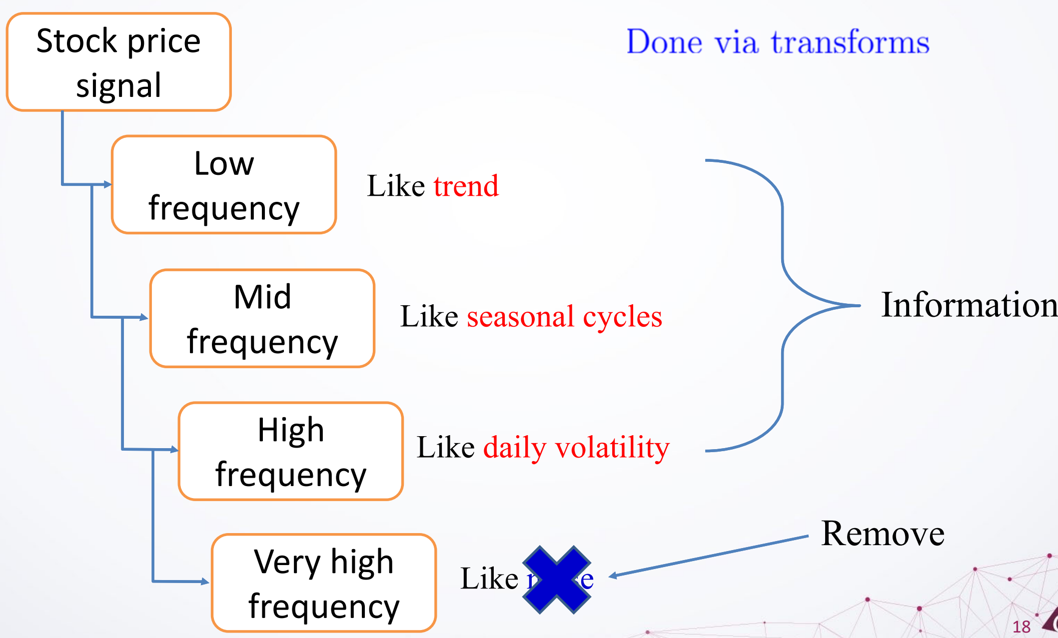
Signal Processing

- A concert hall with different instruments playing
- Each instrument is playing a single note continually
- Can you tell them apart?

- What if we have noise?

Multi-resolution Analysis

Done via transforms



Haar's Wavelet Transform (1909)

Consider the 16 numbers:

$$I = \{1, 2, 3, 4, 5, 6, 7, 8, 8, 7, 6, 5, 4, 3, 2, 1\}.$$

Add neighboring two numbers:

$$S_1 = \{3, 7, 11, 15, 15, 11, 7, 3\}$$

and subtract neighboring two:

$$D_1 = \{-1, -1, -1, -1, 1, 1, 1, 1\}.$$

$I = S_1 \oplus D_1$

Repeat the same procedure for S_1:

$$S_2 = \{10, 26, 26, 10\}$$

$$D_2 = \{-4, -4, 4, 4\}$$

$S_1 = S_2 \oplus D_2$

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Haar's Wavelet Transform (1909)

Repeat the same procedure for S_2:

$$S_3 = \{36, 36\}, D_3 = \{-16, 16\}$$

$S_2 = S_3 \oplus D_3$

Finally, we have:

$$S_4 = \{72\}, D_4 = \{0\}$$

$S_3 = S_4 \oplus D_4$

Notice that:

$$I = D_1 \oplus S_1$$

The Haar's transform of $\{1, 2, 3, 4, 5, 6, 7, 8, 8, 7, 6, 5, 4, 3, 2, 1\}$ is:

$$W = \{\underbrace{-1, -1, -1, -1, 1, 1, 1, 1}_{D_1}; \underbrace{-4, -4, 4, 4}_{D_2}; \underbrace{-16, 16}_{D_3}; \underbrace{0}_{D_4}; \underbrace{72}_{S_4}\}.$$

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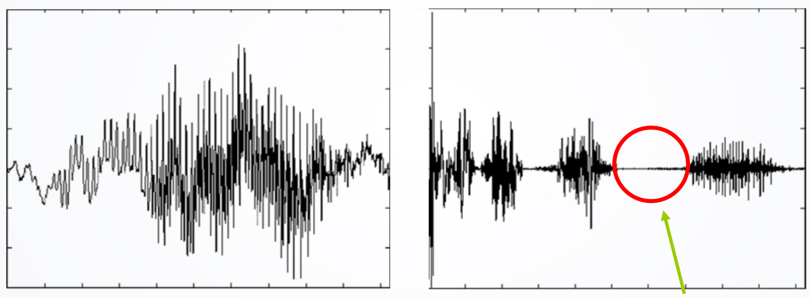
Realistic Signal

Original signal | Wavelet transform coefficients



2048 data | $400\ D_i \approx 0$

- Noise lives at where $D_i \approx 0$
- Setting $D_i = 0$ eliminates the noise

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Hang Seng Index from January 2007 to December 2011

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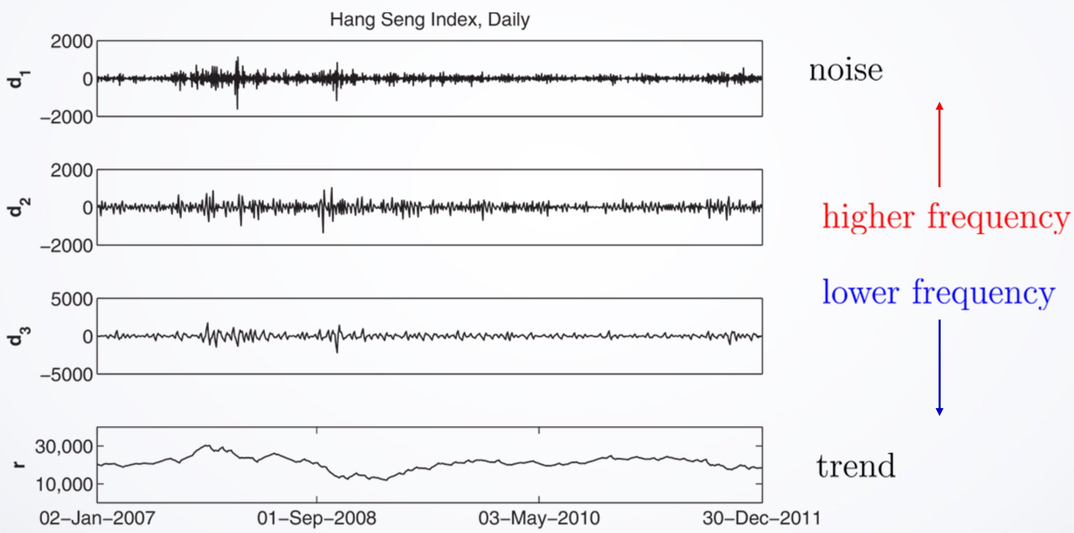
Wavelet Analysis of Hang Seng Index from Jan 07 to Dec 11



$$f(t) = d_1(t) + d_2(t) + d_3(t) + r(t)$$

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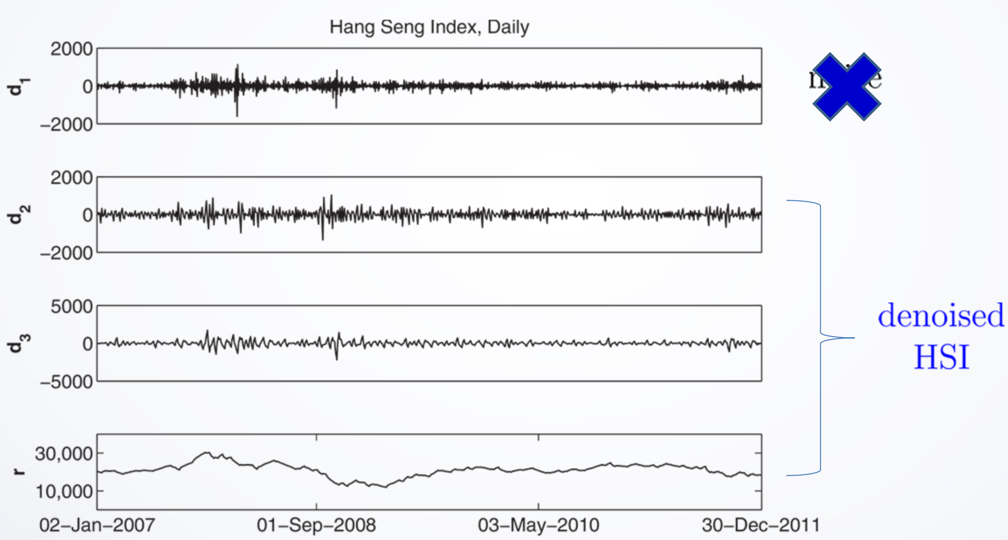
Wavelet Analysis of Hang Seng Index from Jan 07 to Dec 11



$$f(t) = d_1(t) + d_2(t) + d_3(t) + r(t)$$

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Multi-resolution Moving Average

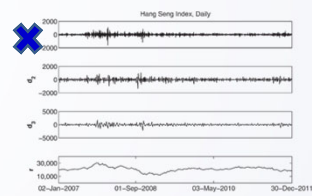
- Stock price up to current day j:

today

$$\mathbf{x}_j = (x_{-n_0}, \ldots, x_{-1}, x_0, x_1, x_2, \ldots, x_j)$$

- Perform wavelet transform on $\mathbf{x}_j$:

$$\mathbf{x}_j = \sum_{k=1}^{m} \mathbf{d}_k^{(j)} + \mathbf{r}^{(j)}, \qquad 1 \le j \le n$$



  - $\mathbf{d}_k^{(j)} = (d_{k,-n_0}^{(j)}, \ldots, d_{k,-1}^{(j)}, d_{k,0}^{(j)}, d_{k,1}^{(j)}, \ldots, d_{k,j}^{(j)})$
  - $\mathbf{r}^{(j)} = (r_{-n_0}^{(j)}, \ldots, r_{-1}^{(j)}, r_0^{(j)}, r_1^{(j)}, \ldots, r_j^{(j)})$

- $\text{MRMA}_{k_1,j_1,j} := \sum_{k=k_1}^{m} d_{k,j-j_1}^{(j)} + r_{j-j_1}^{(j)}$

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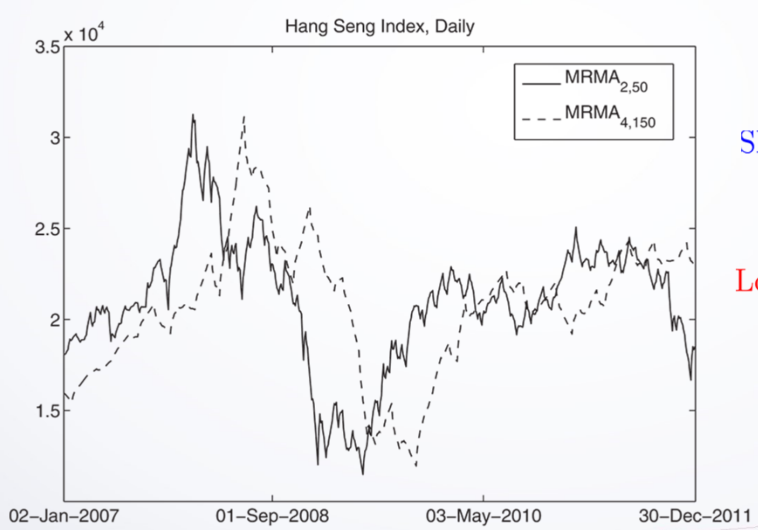
Long and Short Lines MRMA

Short and long lines MR-Moving Averages for Hang Seng Index from 2007 to 2011



Short: noisier

Long: smoother but delayed

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MRMA Trading Rule

- Short line MRMA with small k_1 and j_1
- Long line MRMA ($k_2 > k_1$, $j_2 > j_1$):

$$\text{MRMA}_{k_2,j_2,j} := \sum_{k=k_2}^{m^{(j)}} d_{k,j-j_2}^{(j)} + r_{j-j_2}^{(j)}$$

- Buy day if:

$$\text{MRMA}_{k_1,j_1,j} > \text{MRMA}_{k_2,j_2,j} \times (1 + p\%)$$

- Sell day if:

$$\text{MRMA}_{k_1,j_1,j} < \text{MRMA}_{k_2,j_2,j} \times (1 - p\%)$$

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Evaluation

- Test on Hang-Seng Index and Shanghai Composite Index from January 2007 to December 2011
  - N_b, N_s: numbers of buy days and sell days
  - μ_b, μ_s: mean daily return on buy and sell signals
  - μ: mean daily return on buy-and-hold strategy
  - T_b: test statistics on $(\mu_\text{b} = \mu)$ against $(\mu_\text{b} > \mu)$
  - T_s: test statistics on $(\mu_\text{s} = \mu)$ against $(\mu_\text{s} > \mu)$
  - T_bs: test statistics on $(\mu_\text{b} = \mu_\text{s})$ against $(\mu_\text{b} \neq \mu_\text{s})$
  - BE: break-even transaction cost

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Return on Hang Seng Index (07-11) using Haar's Wavelet

daily return 0.001 ↓ annual return 25.2%

1-sided stat
90% → 1.28
95% → 1.65
99% → 2.33

2-sided stat
90% → 1.65
95% → 1.98
99% → 2.58

Rule (k_1, j_1, k_2, j_2, p)	N_b	N_s	$\hat{\mu}_b$ (T_b)	$\hat{\mu}_s$ (T_s)	$\hat{\mu}_b - \hat{\mu}_s$ (T_{bs})	BE
(1, 0, 5, 150, 0%)	741	492	0.00060 (0.80)	−0.00110 (−0.79)	0.00170 (1.30)	0.0411
(1, 0, 5, 150, 1%)	716	475	0.00057 (0.76)	−0.00108 (−0.76)	0.00165 (1.23)	0.0440
(2, 15, 5, 150, 0%)	732	501	0.00058 (0.77)	−0.00104 (−0.75)	0.00162 (1.26)	0.0590
(2, 15, 5, 150, 1%)	714	487	0.00048 (0.65)	−0.00115 (−0.83)	0.00164 (1.24)	0.0533
(3, 50, 5, 150, 0%)	699	534	0.00063 (0.81)	−0.00100 (−0.76)	0.00163 (1.31)	0.0609
(3, 50, 5, 150, 1%)	679	506	0.00068 (0.87)	−0.00116 (−0.85)	0.00184 (1.41)	0.0748

efficient market

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China Shanghai Composite Index (07-11) using Haar's Wavelet

daily return 0.001 ↓ annual return 25.2%

1-sided stat
90% → 1.28
95% → 1.65
99% → 2.33

2-sided stat
90% → 1.65
95% → 1.98
99% → 2.58

Rule (k_1, j_1, k_2, j_2, p)	N_b	N_s	$\hat{\mu}_b$ (T_b)	$\hat{\mu}_s$ (T_s)	$\hat{\mu}_b - \hat{\mu}_s$ (T_{bs})	BE
(1, 0, 3, 50, 0%)	586	629	0.00157 (1.77)	−0.00180 (−1.59)	0.00337 (2.92)	0.0684
(1, 0, 3, 50, 1%)	563	617	0.00153 (1.73)	−0.00200 (−1.77)	0.00354 (3.03)	0.0778
(1, 0, 4, 100, 0%)	649	566	0.00112 (1.38)	−0.00166 (−1.38)	0.00278 (2.37)	0.0596
(1, 0, 4, 100, 1%)	621	546	0.00112 (1.34)	−0.00178 (−1.46)	0.00290 (2.40)	0.0557
(2, 15, 4, 100, 0%)	655	560	0.00121 (1.45)	−0.00179 (−1.52)	0.00300 (2.56)	0.0598
(2, 15, 4, 100, 1%)	624	540	0.00141 (1.62)	−0.00172 (−1.42)	0.00313 (2.59)	0.0646

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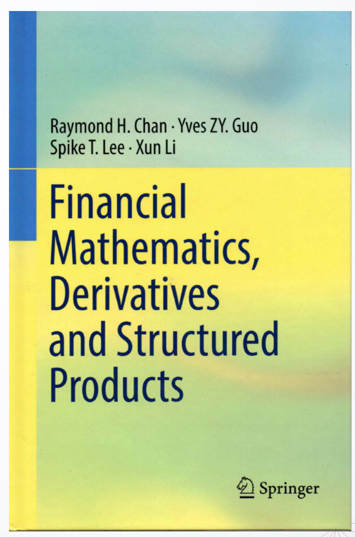
Conclusion

- Wavelet is a data-adaptive denoising tool that can keep more information after filtering out the noise
- The denoised market prices can give more accurate information about price movements
- There are many filters to remove noises (e.g., Empirical Mode Decomposition)
- Using mathematics, we can design new and profitable indicators

Technical Analysis vs Arbitrage Trading

When is a random walk
too random?

- *Black-Scholes Equations* published in 1973, obtained 1993 Nobel Prize
- *2011 Lecture Notes on Black-Scholes Equations* by R.H. Chan on internet
  (Top 5 on Hong Kong Google search for "Black-Scholes" for some years now)
- *Financial Mathematics, Derivatives and Structured Products*, R.H. Chan, Y. Guo, S. Lee, and X. Lee, 347pp., Springer, 2019
  (80,000+ paid downloads on Springerlink)



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Bachelor of Science in **Computing Mathematics @ CityU**

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BSc Computing Mathematics (JS1206)

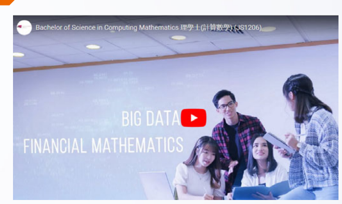
Features

- Data Analysis
- Mathematical Modelling
- Probability & Statistics
- Scientific Computing
- Technical Computer Software

New streams:
- **Enriched Mathematics**
- **Financial Mathematics**

Electives

- Actuarial Science
- Computation Geometry
- Functional Analysis
- Machine Learning
- Mathematical Finance
- Sampling Survey Methods
- Stochastic Processes



BSCM Curriculum

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Bachelor of Laws & Bachelor of Science in Computing Mathematics 法律學學士與理學士（計算數學）雙學位課程 (JS1220)

- The **five-year double degree programme** adopts an integrated curriculum
- 1st - 4th years tuition fees funded by UGC; 5th year on self-financing basis

Highlights:

- Provide legal specialisations with analytical skills such as probability, statistics, quantitative analysis, and logic that are important in the legal world
- Address market needs in intellectual property (IP) and legal serving industries
- Finance, IT, medical, start-up institutions need data-oriented IP and legal services

LLB/BSc Curriculum

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Joint Bachelor's Degree Programmes

- For Mathematics major students with outstanding academic performance
- **2 years at CityU + 2 years at partner university**
- TWO Bachelor's Degrees upon graduation: one from CityU + one from partner university

City U COLUMBIA
City University of Hong Kong | Columbia University Joint Bachelor's Degree Program

THE UNIVERSITY of EDINBURGH

Programme Details

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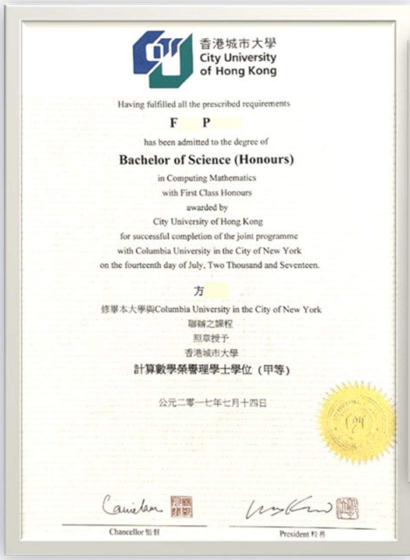
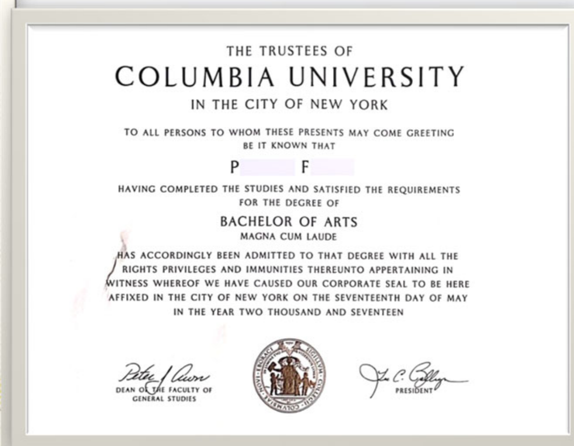
Two Graduate Certificates Upon Graduation

CityU

Columbia University





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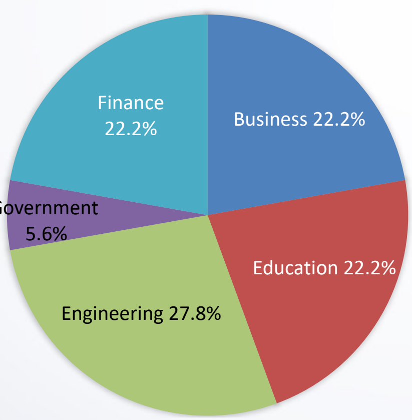
Career Opportunities

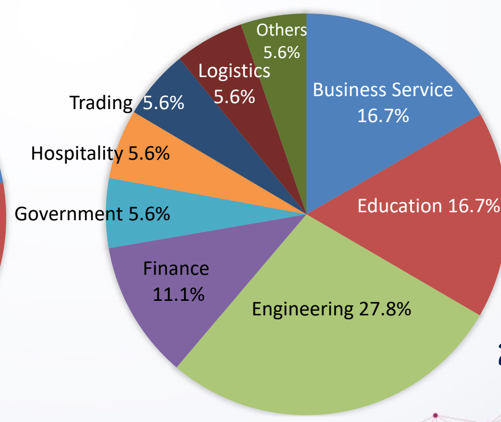
Industries and Firms

- Data Analytics
- Logistics & communications
- Mathematical Modeling
- Quantitative Business Planning
- Scientific Programming
- Teaching (Secondary / Post-secondary)
- Techno-startup

HSBC
Goldman Sachs
Standard Chartered
Societe Generale
DHL



2020



2021

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